Stability of Liquid Fuel Rocket Engine Operation

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In view of the nature of working process in the combustion chamber and of the fuel-feed system, the liquid fuel rocket engine can be regarded as an oscillatory system with a delayed feedback. The unsteady processes in the fuel mains are described by a wave equation. The delay and the wave processes condition the alternation of steady and unsteady regimes as the thrust diminishes. The frequencies of oscillation occurring at that time are near the natural frequencies of the fuel-pipe run. Besides, for great delays, only relatively low frequency oscillating regimes are possible.

In an analysis of rocket engine stability, the author derives equations for the combustion chamber, for the fuel-feed system, and for the process of transformation of liquid fuel into gaseous combustion products. A characteristic equation for the entire system is then derived and used for plotting the limits of the region of stable performance of the system and for calculating the oscillation frequency at the stability region limits.

1 Liquid Fuel Rocket Engine as a Natural Oscillating System

THE SELF-EXCITATION of oscillations in a liquid fuel rocket engine is to a significant degree determined by the character of the transformation process of liquid fuel components into gaseous combustion products. Prior to their transformation into gaseous products, both the oxidizer and the fuel must pass through a stage of preparatory processes (pulverization of liquid jets, heating and evaporation of drops, mixing of components, chemical reactions, etc.). The fuel mass passing through that stage does not practically add gaseous products, whose inflow determines the pressure in the chamber. Under conditions of fast variation of fuel consumption, this results in the practicability of determining the pressure in the chamber at a given moment by the values of fuel consumed during a certain preceding time interval. In other words, the relationship between fuel consumption at the rocket's nozzle and the pressure in the chamber is that of lag. For the existing liquid fuel rocket engine's fuel-feed system, this relationship is bilateral. The oscillation of pressure in the chamber also causes a variation in the fuel feed into the chamber. Therefore, by the very nature of its working process and of the fuel-feed system, the liquid fuel rocket engine is an oscillating system with a delayed feedback.

One of the engine's parts—the main fuel pipe—must be regarded as a component with distributed parameters, since the time of pressure wave traveling along the pipe and the period of occurring oscillations are commensurable.

The lagging and the wave processes determine a series of peculiarities of liquid fuel engine dynamics which will be reviewed in this paper by an example of a simple installation.

2 Equations of Liquid Fuel Rocket Engine Dynamics

In order to deduce the stability conditions, we shall write the equations describing the unsteady processes in the liquid fuel rocket engine. As usual when investigating stability in the small, we shall use linearized equations.

Transformation Process Equations

We shall characterize the transformation process of liquid fuel into gas by a transformation curve $\varphi(t)$—ratio of the gas mass formed at the moment t by the combustion of the examined fuel volume to the initial mass of fuel which entered at a time $t = 0$.

The transformation curve obviously ascends smoothly from zero and tends asymptotically toward unity (Fig. 1). This curve characterizes the processes only as an average, its form being determined by a transformation law of a separate particle (drop) of fuel, as well as by the difference of transformation conditions for various particles (for example, because of nonidentical size of drops). Let us admit that the transformation curve is determined only by steady parameters and that it does not vary at small oscillations.

If we introduce the transformation velocity $\psi = d\varphi/dt$, the mass of gas formed during a time unit $M_r(t)$ will be expressed by the formula

$$M_r(t) = \int_0^\infty M_\Phi(t - t_1)\psi(t_1)dt_1 \qquad [2.1]$$

where $M_\Phi(t)$ is the mass velocity of fuel input through the head of the engine's nozzle. Let us introduce the dimensionless small deflections

$$m_r = \frac{M_r - M_{r0}}{M_{r0}} \qquad m_\Phi = \frac{M_\Phi - M_{\Phi 0}}{M_{\Phi 0}}$$

We shall designate by the index 0 the stationary value of the functions. Taking into account that

$$\int_0^\infty \psi(t_1)dt_1 = 1$$

we shall have in place of [2.1]

$$M_r(t) = \int_0^\infty m_\Phi(t - t_1)\psi(t_1)dt_1 \qquad [2.2]$$

Combustion Chamber Equation

Let us consider that the gas pressure along the whole chamber volume is identical at any given moment. This is valid for oscillations, the period of which is much greater than the time of pressure wave run of the chamber's length.

The equation of gas mass preservation in the chamber has the form

$$dQ_k/dt + M_c = M_r \qquad [2.3]$$

where Q_k is the gas mass in the chamber and M_c is the mass velocity of gas outflow through the supersonic nozzle.

If the correlation of fuel components is not varying at oscillations, the processes in the chamber may be considered isentropic, i.e.

$$p_k\rho_k^{-\gamma} = \text{const} \qquad [2.4]$$

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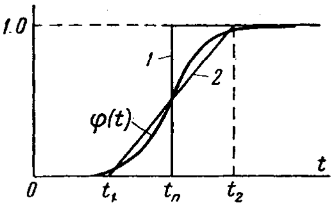


Fig. 1

To Eqs. [2.4] and [2.5] should be added the equation of the state of an ideal gas:

$$p_k = \rho_k R T_k$$

and the expressions for Q_k and M_c:

$$Q_k = \rho_k V_k \qquad M_c = \frac{p_k F_{kp}}{g\beta}$$

$$\beta = \frac{\sqrt{\gamma R T_k}}{\gamma}\left(\frac{\gamma+1}{2}\right)^{(\gamma-1)/2(\gamma+1)} \qquad [2.5]$$

where p_k, ρ_k, and T_k are, respectively, the pressure, density, and temperature of the gases in the combustion chamber, R the gas constant, γ the adiabatic index, V_k the volume of the combustion chamber, F_{kp} the surface of the nozzle's critical section, and g the gravity acceleration.

We shall obtain from Eqs. [2.3–2.5] the relationship between m_r and the dimensionless pressure variation in the chamber

$$\eta_k = \frac{p_k - p_{k0}}{p_{k0}}$$

in the following form

$$t_k \frac{d\eta_k}{dt} + \chi\eta_k = m_r \quad \left(t_k = \frac{Q_{k0}}{\gamma M_{r0}},\ \chi = \frac{\gamma+1}{2\gamma}\right) \qquad [2.6]$$

Here t_k is the time of presence of gases in the combustion chamber, divided by the adiabatic index.

Equations of the Fuel-Feed System

It is assumed that the forces of inertia exert no effect on liquid flow, these forces occurring at time of rocket acceleration. Let us examine a fuel-feed system by compressed gas, consisting of a fuel tank, of a uniform piping, and with a nonelastic nozzle head.

The motion of the liquid in the cylindrical straight pipe is defined by Zhukowski equations

$$-\frac{\partial p}{\partial x} = \rho\frac{\partial v}{\partial t} \qquad -\frac{\partial p}{\partial t} = \rho a^2 \frac{\partial v}{\partial x} \qquad [2.7]$$

In these equations ρ is the liquid's constant density, and a is the velocity of sound in the liquid located in the elastic pipe.

The tank end of the piping may be considered open, and the pressure in this section is constant at oscillations:

$$x = 0 \qquad \frac{\partial v}{\partial x} = 0 \qquad [2.8]$$

Neglecting the liquid's inertia in the injector head cavity, we shall obtain a second boundary condition from the expression for fuel consumption through the nozzle:

$$x = l \qquad \rho v = \mu_\Phi \sqrt{p - p_k} \qquad [2.9]$$

Introducing the variables

$$m = \frac{\rho v - \rho v_0}{\rho v_0} \qquad \eta = \frac{p - p_0}{p_{k0}}$$

and eliminating η from [2.7], we shall obtain for m the wave equation

$$\frac{\partial^2 m}{dt^2} - a^2\frac{\partial^2 m}{\partial x^2} = 0 \qquad [2.10]$$

with boundary conditions:

$$\frac{\partial m}{\partial x_0} = 0 \quad \text{when} \quad x = 0$$

$$m = \frac{\eta - \eta_k}{h_\Phi} \quad \text{when} \quad x = l \qquad [2.11]$$

$$h_\Phi = \frac{2[p_0(l) - p_{k0}]}{p_{k0}} = \frac{2\Delta p_\Phi}{p_{k0}}$$

The second condition [2.11] is obtained by the linearization of Eq. [2.9].

3 Characteristic System Equation

Let us apply to the obtained equations the Laplace transformation, keeping for the transformed equations the former designations. Let us designate by q the differential operator.

The wave equation [2.10] will then become an ordinary differential equation, the solution of which for $m(q, l)$ equal to m, taking account of boundary conditions [2.11], will be obtained in the form

$$m_\Phi = -\frac{\eta_k}{h_\Phi + h_b \tanh \dfrac{ql}{a}} \quad \left(h_b = \frac{\rho v_0 a}{p_{k0}}\right) \qquad [3.1]$$

Instead of [2.1] and [2.7], we shall obtain

$$m_r = \psi(q) m_\Phi \qquad [3.2]$$

$$\eta_k = \frac{m_r}{\chi + q t_k} \qquad [3.3]$$

It is easy to obtain from Eqs. [3.1–3.3] the system's characteristic equation

$$h_\Phi + h_b \tanh\frac{ql}{a} + \frac{\psi(q)}{\chi + q t_k} = 0 \qquad [3.4]$$

4 Stability Boundary of Liquid Fuel Rocket Engine's Operation

We shall obtain the equations for the delimitation of boundaries of liquid fuel rocket engine's stable operation and for the computation of oscillation frequency ω at stability boundary by substituting in [3.4]

$$q = i\omega \qquad (i = \sqrt{-1})$$

These equations may be brought to the form:

$$h_\Phi^2 = \frac{S^2(\omega)}{\chi^2 + \omega^2 t_k^2} h_b^2 \tan^2\alpha \qquad \left(\alpha = \frac{\omega l}{a}\right) \qquad [4.1]$$

$$\tan\theta = -\frac{h_\Phi \omega t_k - \chi h \tan\alpha}{-\chi h_\Phi + \omega t_k h_b \tan\alpha} \qquad [4.2]$$

Here

$$S^2(\omega) = \mathrm{Re}^2\psi(i\omega) + \mathrm{Im}^2\psi(i\omega) \qquad \tan\theta = \frac{\mathrm{Im}\psi(i\omega)}{\mathrm{Re}\psi(i\omega)}$$

The frequency functions $S(\omega)$ and $\theta(\omega)$ are, respectively, the amplitude and the phase spectra of the curve $\psi(y)$.

For a subsequent analysis it is necessary to assign the transformation curve a concrete form. In many cases, the curve $\varphi(t)$ may be approximated by a stepwise function (curve 1 of Fig. 1):

$$\varphi = 0 \qquad 0 < t < t_n \qquad \varphi = 1 \qquad t \geqslant t_n$$

This implies that the fuel is instantaneously transformed into gas t_n sec after entering the combustion chamber. For such a curve

$$S(\omega) = 1 \qquad \theta = -\omega t_n$$

By approximating $\varphi(t)$ by function 2 (Fig. 1), we shall obtain

$$S^2(\omega) = \left[\frac{\sin(\omega t_2/2)}{\omega t_2/2}\right]^2 \qquad \theta = -\omega\left(t_1 + \frac{t_2}{2}\right)$$

For curve 2, just as for any symmetric curve $\psi(t)$, $\theta = -\omega t_n$, where the delay time t is the abscissa of the symmetry axis.

We shall examine with more detail the case of a stepwise transformation curve. If we introduce for that case the magnitude l/a as a time scale, we have in place of [4.1] and [4.2]:

$$h_\Phi = \sqrt{\frac{1}{\chi^2 + \alpha^2\tau_k^2} - h_b^2\tan^2\alpha} \qquad [4.3]$$

$$\tan\alpha\tau = \frac{h_\Phi\alpha\tau_k - \chi h_b\tan\alpha}{-\chi h_\Phi + \alpha\tau_k h_b\tan\alpha} \qquad [4.4]$$

$$\text{sign}\sin\alpha\tau_n = \text{sign}(h_\Phi\alpha\tau_k - \chi h_b\tan\alpha)$$

$$\left(\chi = \frac{\gamma+1}{2\gamma} \qquad \alpha = \frac{\omega l}{a} \qquad \tau_n = \frac{t_n l}{a} \qquad \tau_k = \frac{t_n l}{a}\right)$$

The stability of the liquid fuel rocket engine's operating regime will be determined by the following four dimensionless parameters, as this follows from [4.3] and [4.4]:

$$\tau_n = \frac{t_n l}{a} \qquad \tau_k = \frac{t_n l}{a} \qquad h_\Phi = \frac{2\Delta p_\Phi}{p_{k0}} \qquad h_b = \frac{\rho v_0 a}{p_{k0}}$$

where τ_n is the ratio of the delay time to the time required by the sound wave to travel the length of the pipe l/a; τ_k is the ratio of time the gas stays in the chamber (reduced γ times) to l/a; h_Φ is the doubled ratio of pressure drop in the nozzles to the pressure in the chamber, and h_b is the ratio of the impact pressure occurring with the instantaneous liquid deceleration to the pressure in the chamber. The parameter

$$h = h_b/h_\Phi = \rho v_0 a/2\Delta p_\Phi$$

appears to be a more descriptive hydraulic parameter. However, in analyzing the stability it is more convenient to use the parameter h_b.

The magnitude $\chi = (\gamma + 1)/2\gamma$ also stands among parameters. However, it may be considered with sufficient precision that kt is equal to the unity.

In the course of the liquid fuel rocket engine's thrust regulation by means of lowering pressure in fuel tanks, the parameters τ_k and h_b remain constant, the parameter h_Φ diminishes proportionally to the pressure in the chamber (and to the thrust), the τ_n variation may be extremely diversified, depending on the kind of fuel used and on the mixing. For most engines, one may estimate that in the course of regulation the delay time increases as the thrust diminishes. Therefore, it is appropriate to plot the stability boundary in the plane h_Φ, τ_n.

Since h_Φ is a real magnitude, limitations on oscillation frequencies follow from Eq. [4.3]: the only possible frequencies are either near $\alpha = 0$ or near $\alpha = n\gamma(9n = 1, 2, \ldots)$. For these frequencies, the reactive resistance of the liquid column (magnitude $h_b\tan\alpha$) is near zero. It is obvious that $\alpha = n\pi$ corresponds to the natural frequencies of the

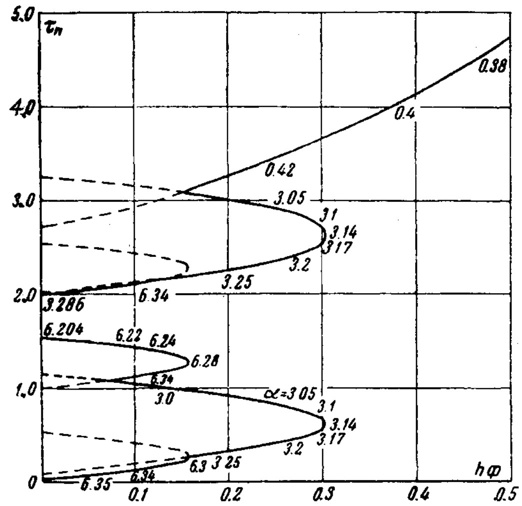


Fig. 2

liquid's acoustic oscillations in the piping, i.e., to frequencies

$$f = n\frac{a}{2l}\ Mc/s$$

The value $n = 1$ corresponds to the basic tune, $n = 2, 3, \ldots$ to its second, third, etc., harmonic. At low frequencies, the function

$$h_b\tan\alpha \approx \omega t_l \qquad (t_l = \rho v_0 l/p_{k0})$$

and the liquid's inertness are determined by a single time constant. It may be said with enough precision that

$$\tan\alpha \approx \omega l/a \qquad \text{when} \qquad \alpha \leqslant \tfrac{1}{6}\gamma$$

Consequently, the liquid's compressibility may not be accounted for until $f \leqslant (a/12l)(Mc/s)$.

On determining h_Φ for the given value from Eq. [4.3], we may find the value at the stability limit from Eq. [4.4]. The magnitude $\alpha\tau_n$ is determined with precision until $2k\gamma$ ($k = 1, 2, \ldots$), i.e., oscillations with the same dimensionless frequency α are possible for several values τ_n, differing from one another by $2\pi/\alpha$.

The total stability boundary in the plane of parameters h_Φ, τ_n for $h_b = 2$ and $\tau_k = 1$, and for the values $n = 1, 2$ and $k = 1, 2$, is plotted on Fig. 2. The stability region is situated to the right, and it is bounded by a smooth upper curve (low frequencies) and a tooth-like curve, for which the oscillation frequencies are near the piping's natural frequencies. The

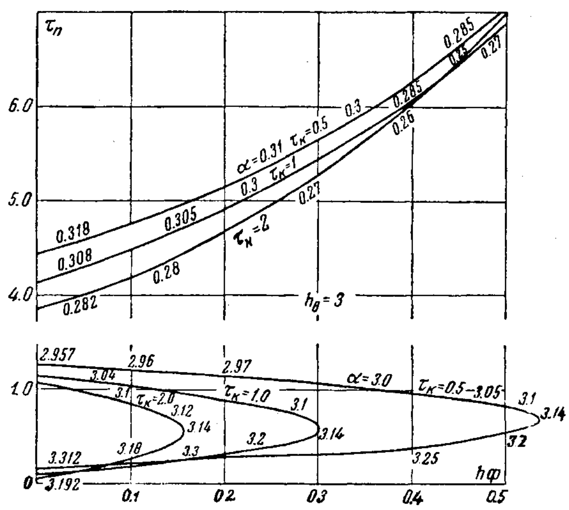


Fig. 3

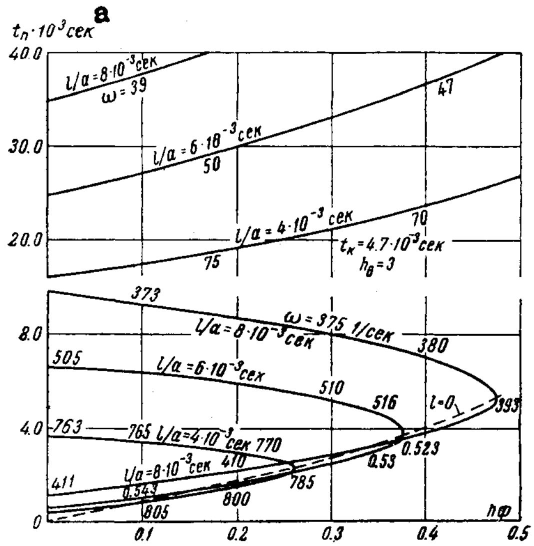


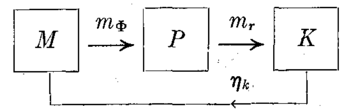
Fig. 4 ^{a} sec

"teeth" corresponding to values $n = 1$, $k > 1$ and $n = 2$, $k > 2$ do not modify the stability region in this case.

The operating point (h_Φ, τ_n) for the given engine is shifting to the left and upward. Consequently, the stable and unstable regimes may alternate as the liquid fuel rocket engine's thrust diminishes. The increase of τ_n will not lead the engine out of the instability region if the operating point passes the upper smooth boundary, which tends to infinity as h nears 1 or, rather more accurately, χ.

The influence of the other parameters on the stability region easily may be followed up by reviewing separately the upper boundary and the tooth with $k = 0$, $n = 1$. The influence of τ_k and h_b may be seen on Figs. 2 and 3. The increase of h_b leads to the narrowing of teeth, without changing either their position or length, increasing at the same time the number of teeth (values k). The influence of dimensional time l/a is illustrated in the plane h_n, t_n in Fig. 4. The influence of l/a on the stability of liquid fuel rocket engine operation may be quite diversified. The increase of l/a may lead to the contraction of the stability region at the expense of a lengthening of teeth and strongly widens it, shifting the upper boundary. As l/a increases, the points of the teeth rise. However, for high values of l/a, it is necessary to account for the teeth corresponding to harmonics of the basic tune. It is interesting to note that with variation of l/a the points of the teeth for $n = 1$ are displaced along a curve corresponding to $l = 0$ (dotted line of Fig. 4.)[1]

Resulting from the preceding, the liquid fuel rocket engine may be represented by the following block diagram:



where the links M, P, K, respectively, designate the fuel-feed system, the transformation process, and the combustion chamber as a gas cubic content. The characteristic equation of such system is written outright according to known transmitting function of the links:

$$K(q)M(q)P(q) = 1$$

For the above-examined engine installation, we obtain from Eqs. [3.1], [3.2], and [3.3]:

$$M(q) = -\frac{1}{h_\Phi + h_b th(ql/z)}$$

$$P(q) = \psi(q)$$

$$K(q) = \frac{1}{\chi + qt_k}$$

For the analysis of the stability of a liquid fuel rocket engine it is necessary to know the foregoing three transmitting functions. For complex fuel-feed systems and for the transformation process, it is advisable to determine experimentally the transmitting functions or the frequency characteristics which may replace the transmitting functions.

[1] This case ($l = 0$, stepwise transformation curve) was examined earlier by Natanzon.

Reviewer's Comment

In view of the indisputable lead of the Soviet Union in the field of large rocket boosters, it seems appropriate to publish this translation, which seems to indicate, with many others, that this lead does not convey a corresponding depth of knowledge about the important basic problems associated with liquid propellant rockets.

The problem treated here has been developed in this country by Sabersky.[2] No reference is given to this paper (or any other Western paper) by the Russian author.

The treatment is presented in a very polished way, following standard mathematical techniques.

—Luigi Crocco
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Princeton University

[2] Sabersky, R. H., "Effect of wave propagation feed lines on low-frequency rocket instability," Jet Propulsion **24,** 172–174 (1954).